Abstract

In this thesis, our main focus is on the decomposition results in terms of sum and product, the construction of the more general fractal function and fractal measure on some fractal domains, the fractal dimensions of the constructed fractal functions, the dimensional results on the fractal transformations, and the quantization dimension for the invariant measures.

Fractal dimension is one of the most important concepts in the fractal geometry. Fractal dimensions are useful to characterize the sets. In this thesis, we prove more general decomposition results of a continuous function as the sum for the packing dimension and the upper box dimension. We also prove decomposition results of a continuous function as the product for the Hausdorff dimension, the lower box dimension, the packing dimension, the upper box dimension and the Assouad dimension. Moreover, we determine upper bound for the upper and the lower box dimension of the graph of the product of two continuous functions.

Fractal interpolation functions are the special type of interpolating function whose graph is a fractal. Generally, these functions are non-smooth and irregular. Due to this, these functions are more suitable for approximation than the classical interpolation functions. In this thesis, we determine the bounds of the Hausdorff dimension of the graph of the vector-valued fractal function. Moreover, we estimate the Hausdorff dimension of the invariant measure supported on the graph of the vector-valued fractal function. We also prove the existence of the more general vector-valued fractal function and the invariant measure supported on the graph of the constructed vector-valued fractal function using the iterated function system (IFS) and Rakotch contraction theory. Furthermore, we determine the fractal dimension of the graph of the constructed fractal functions using the function space technique. Next, we prove the existence of the more general fractal function on the Sierpiński Gasket (SG). We analyse some properties and determine some dimensional results for these fractal functions. We establish some conditions under which these fractal functions have finite energy. After that, we discuss

about the restrictions of these fractal functions on the bottom line segment of SG and determine the fractal dimensions of the graph of the Riemann-Liouville fractional integral of these fractal functions. We also determine the exact value of the fractal dimensions of the graph of the restriction of any harmonic function on SG. After that, we prove the existence of the general version of α-fractal function. We determine the exact value of the fractal dimension of graph of the general version of α-fractal function using covering method. Furthermore, we prove the existence of the fractal function for general data sets using countable IFS and weak contraction theory. We also estimate the fractal dimensions of the graph of the constructed fractal functions.

Fractal transformation is a map between the attractor of one hyperbolic IFS to the attractor of another hyperbolic IFS. It has an application in image synthesis. In the second last chapter of this thesis, we determine some results on the fractal dimension of the graph of the fractal transformations. Quantization dimension is a very important thing in the quantization theory. It has very good relationship with the Hausdorff dimension and the box dimension. The quantization dimension of a Borel probability measure measures the speed how fast the quantization error goes to zero as n goes to ∞. In this thesis, we determine the bounds for the quantization dimension of the invariant measure supported on the attractor of the bi-Lipschitz IFS. We also determine the quantization dimension of the invariant measures supported on the graph of the fractal transformation.